Wave Interactions in a 2-D Left-Handed Structure

Anthony Lai[1], Christophe Caloz[2], and Tatsuo Itoh[1]

[1]Electrical Engineering Department, University of California, Los Angeles, USA

[2]École Polytechnique de Montréal, Québec, Canada

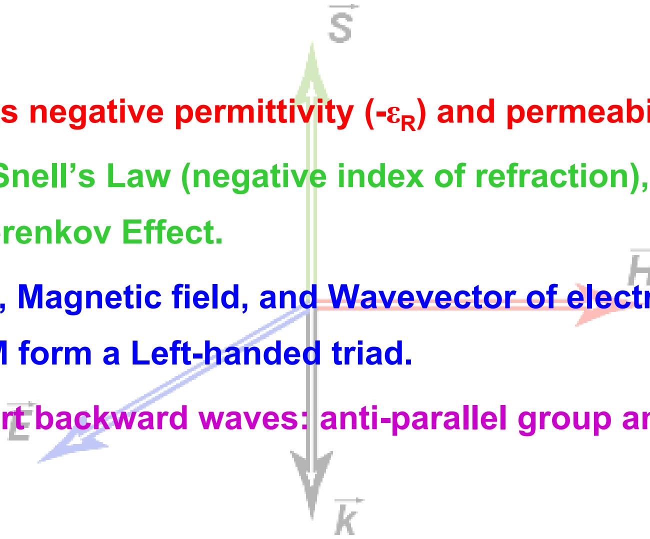
ÉCOLE POLYTECHNIQUE MONTRÉAL

Outline

- Left-Handed Materials (LHMs) Background
- Mushroom Structure as an Effective Composite Right/Left-Handed (CRLH) Surface (PRI/NRI) $\rightarrow$ ε, μ; n
- Mushroom Characteristics – Fields, Refractive Index, Isotropy
- Mushroom Design
- Parabolic Refractor

Background (1)

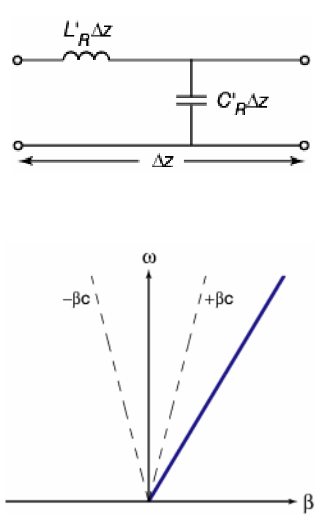
Left-Handed Materials



- Simultaneous negative permittivity ($-\varepsilon_R$) and permeability ($-\mu_R$).
- Reversal of Snell's Law (negative index of refraction), Doppler Effect, and Cerenkov Effect.
- Electric field, Magnetic field, and Wavevector of electromagnetic wave in a LHM form a Left-handed triad.
- LHMs support backward waves: anti-parallel group and phase velocity.

Background (2)

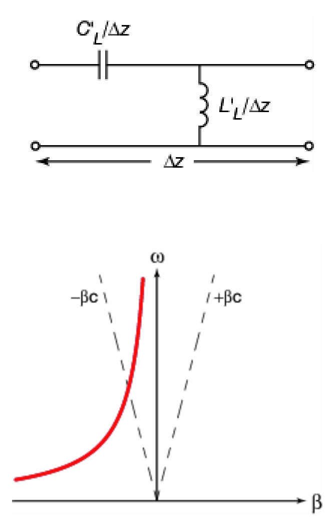
Right-Handed



$$\left(v_p = \frac{\omega}{\beta}\right) \bullet \left(v_g = \frac{d\omega}{d\beta}\right) > 0$$

+

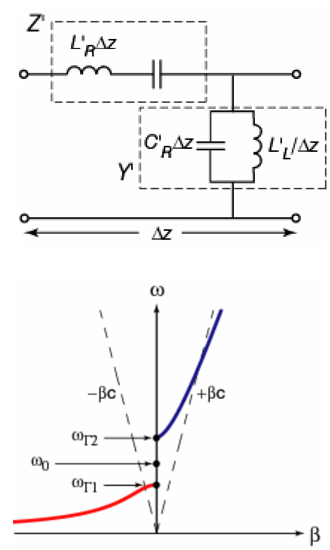
Left-Handed



$$v_p \bullet v_g < 0$$

=

Composite Right/Left-Handed



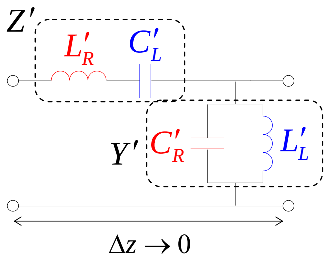
$\omega < \omega_0$: $v_p \bullet v_g < 0$

$\omega > \omega_0$: $v_p \bullet v_g > 0$

- LHMs support backward waves: anti-parallel group and phase velocity; energy travels away from the source, but wavefronts travel backwards toward the source.

2-D Left-Handed Material (1)

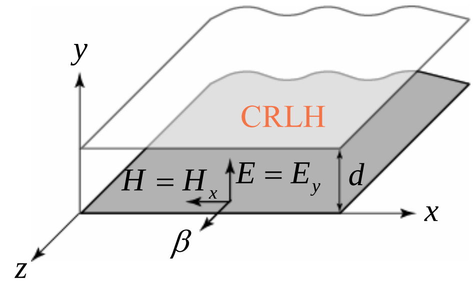
CRLH TL



$$jB = \sqrt{Z'Y'}$$

$$Z_o = \sqrt{\frac{Z'}{Y'}}$$

TEM CRLH PPWG



$$B = \omega\sqrt{\mu\varepsilon}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

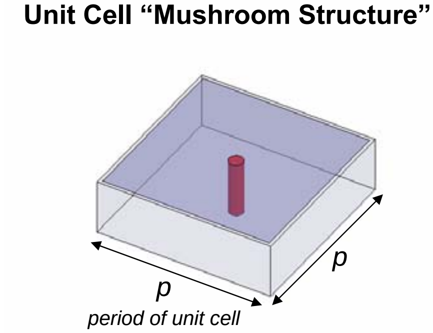
Transmission line relations related to constitutive parameters of CRLH material.

$$\mu = L'_R - \frac{1}{\omega^2 C'_L} \quad \text{and} \quad \varepsilon = C'_R - \frac{1}{\omega^2 L'_L} \Rightarrow n(\omega) = c\sqrt{\varepsilon\mu}$$

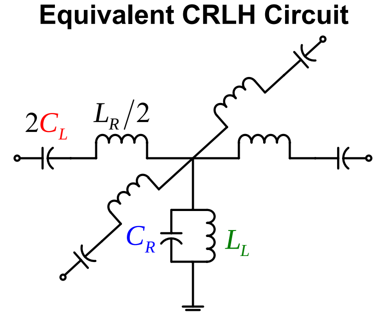
2-D Left-Handed Material (2)

• Distributed Implementation of a 2-D Left-Handed Material

Unit Cell "Mushroom Structure"



Equivalent CRLH Circuit

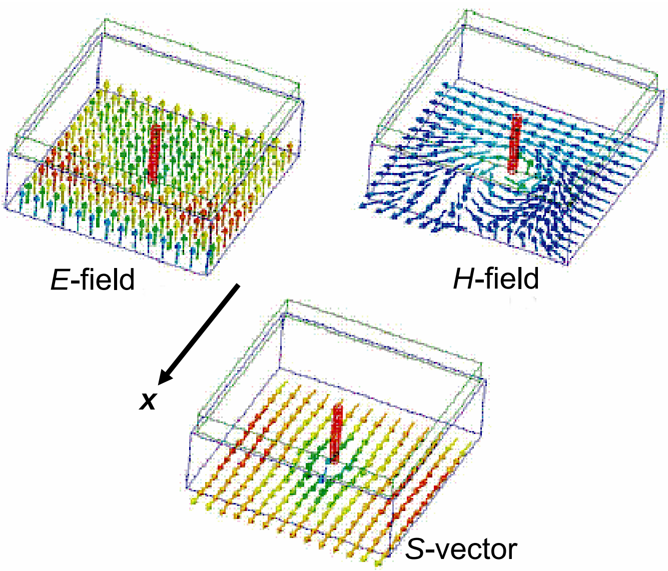


- **Left-handed Capacitance (C_L): Due to mutual capacitive couplings.**
- **Left-handed Inductance (L_L): Due to via to ground plane.**
- **Right-handed Capacitance (C_R): Due to capacitor-like nature of unit cell.**
- **Right-handed Inductance (L_R): Due to current flow on top patch.**

*** Electromagnetic waves "see" the unit cell as effectively homogenous in the long λ region, i.e.** $p < \lambda/4$

Mushroom Characteristics (1)

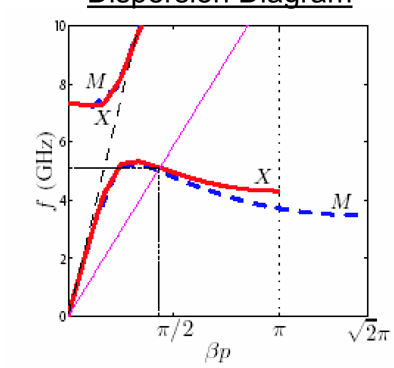
Field Distribution (LH Region)



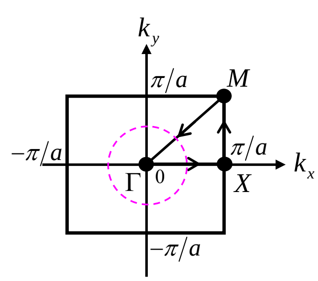
- Quasi-TEM field distribution.
- *H*-field is locally circulating around via to provide LH shunt inductance.
- On average, *H*-field is perpendicular to *E*-field.

Mushroom Characteristics (2)

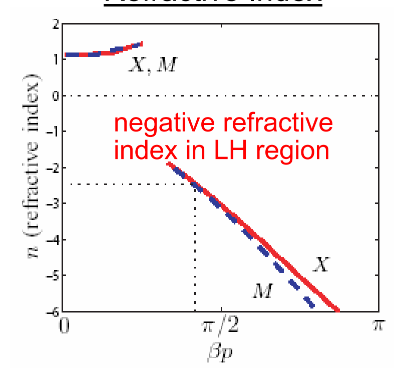
Dispersion Diagram



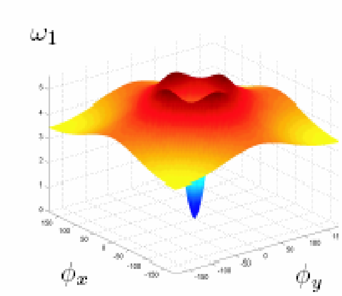
Brillouin Zone

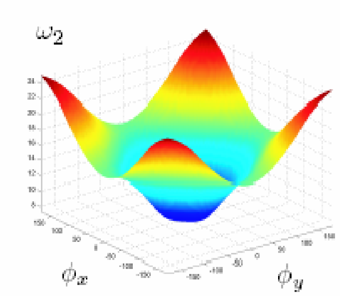


Refractive Index



Isotropicality





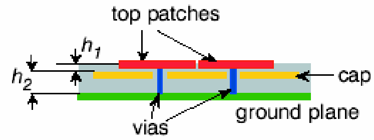
$$\beta(\omega) = \sqrt{k_x^2 + k_y^2}$$

Mushroom is nicely isotropic close to the spectral origin.

Mushroom Improvement (1)

Ways to Increase Left-Handed Attributes of Mushroom Structure

- **Caps** to increase capacitor couplings from adjacent cells (Sanada, Caloz, & Itoh).
  - **Pros**: LH mode occurs at lower frequency; LH-ness occurs closer to Γ- point → more homogeneous.
  - **Cons**: Difficult to implement.



- **Increase substrate height**.
  - **Pros**: Dispersion curve shifted lower.
  - **Cons**: Need to dramatically increase height to see small improvement.

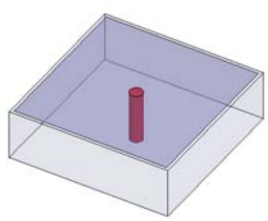
- **Thinner Via**.
  - **Pros**: LH slope is improved; dispersion curve shifted lower.
  - **Cons**: Need a very thin via (radius ~ 0.1 mm) to obtain improvement; resistance increases → lossy; smallest available drill bit has radius of 0.175 mm.

- **Increase permittivity (ε_R) of host substrate**.*
  - **Pros**: Similar affect as adding caps without the need for difficult implementation.
  - **Cons**: High permittivity substrate is more expensive.

* It may appear that increasing the permittivity will dramatically increase C_R and therefore decrease the LH attributes of the mushroom structure, but it appears that C_L is increased significantly and dominates at lower frequency.

Mushroom Improvement (2)

Increase permittivity (ε_R) of host substrate

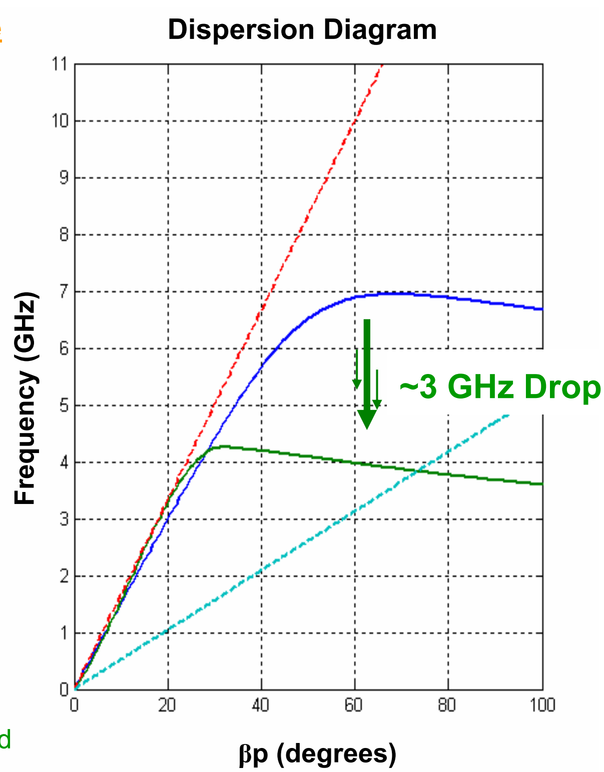


Dimension of Unit Cell

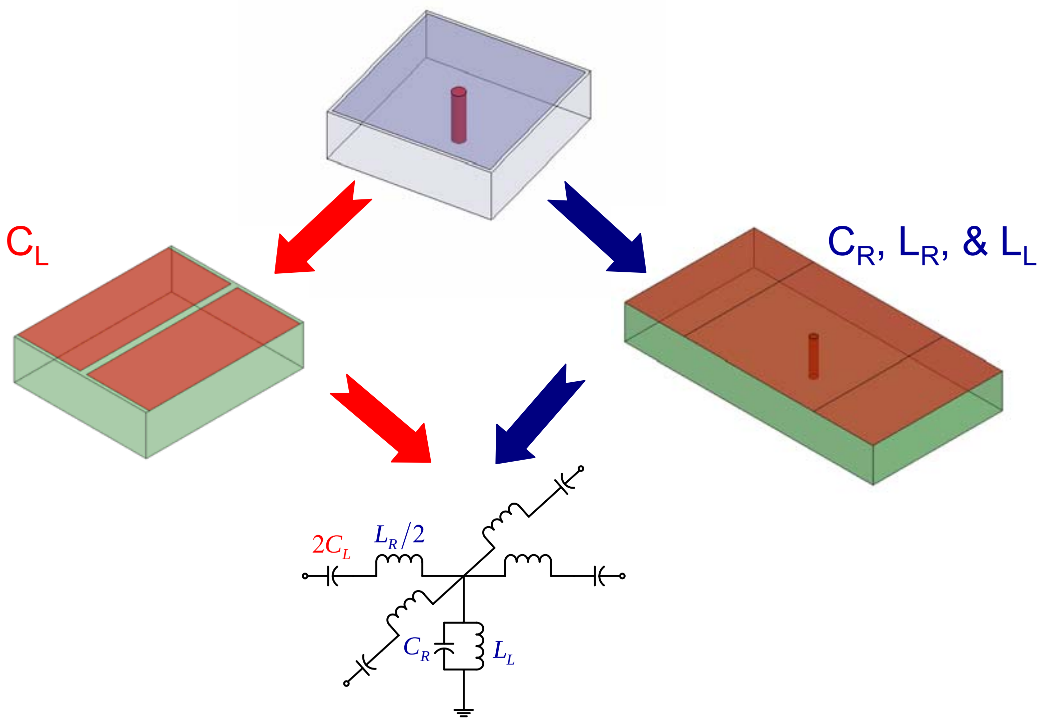
period : 5 mm
top: 4.8 x 4.8 mm^2
via diameter: 0.24 mm

Blue	**Green**
height: 1.57 mm	height: 1.27 mm
ε_R=2.2	ε_R=10.2

*Further improvement can be accomplished by increasing height and thinner via.

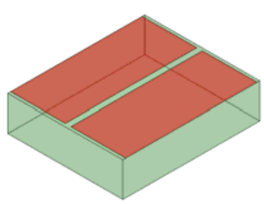


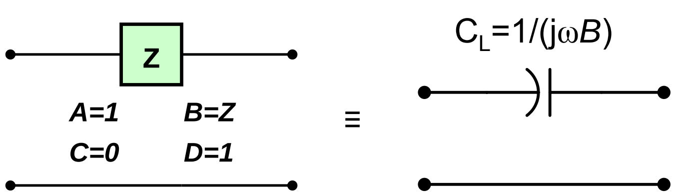
Parameter Extraction (1)



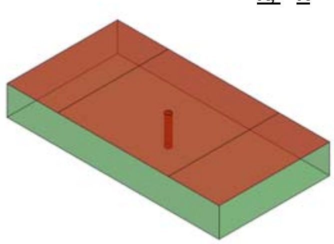
Parameter Extraction (2)

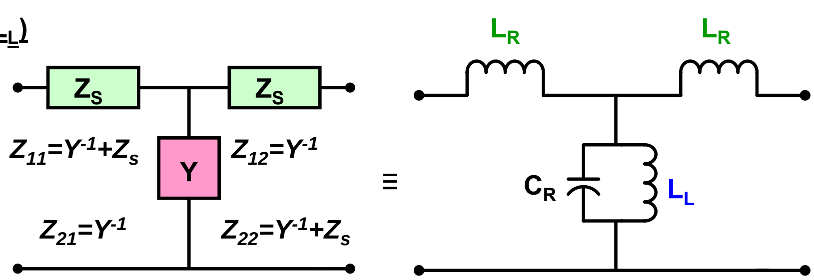
LH Capacitance (C_L)





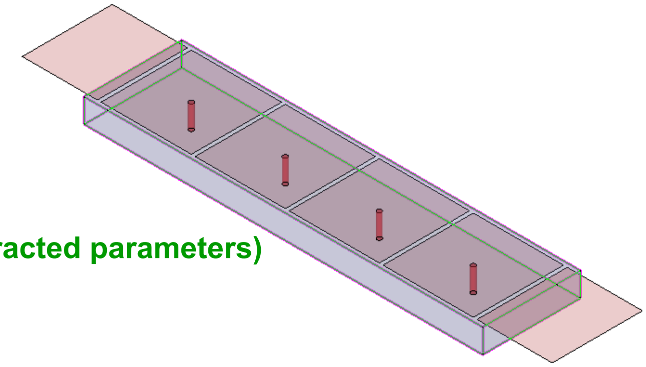
RH & LH Values (C_R, L_R, & L_L)





$$C_R = \frac{1}{2j\omega}\left(\omega\frac{\partial Y}{\partial \omega} + Y\right) \qquad L_R = \frac{Z_{11} - Z_{21}}{j\omega} \qquad L_L = \frac{2j}{\omega}\left(\omega\frac{\partial Y}{\partial \omega} - Y\right)^{-1}$$

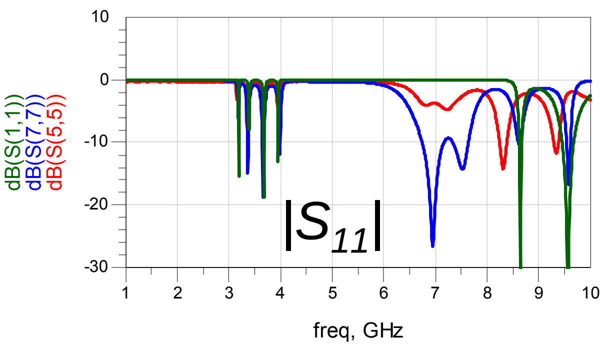
Parameter Extraction (3)

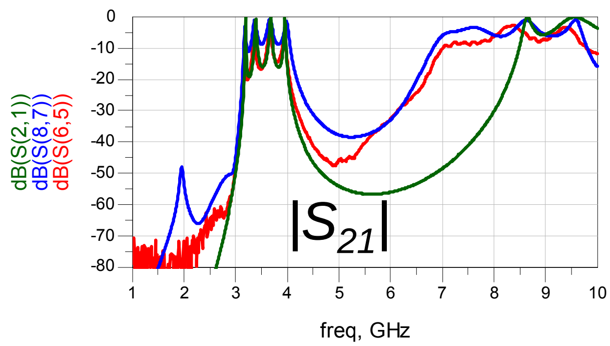


ADS --- (with extracted parameters)

HFSS ---

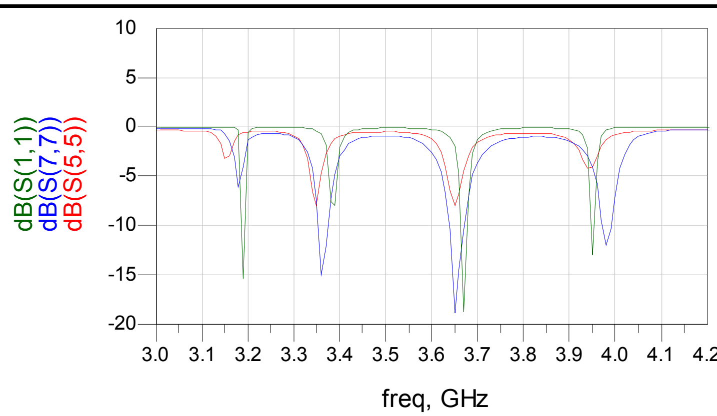
Measured ---





Parameter Extraction (4)

$|S_{11}|$

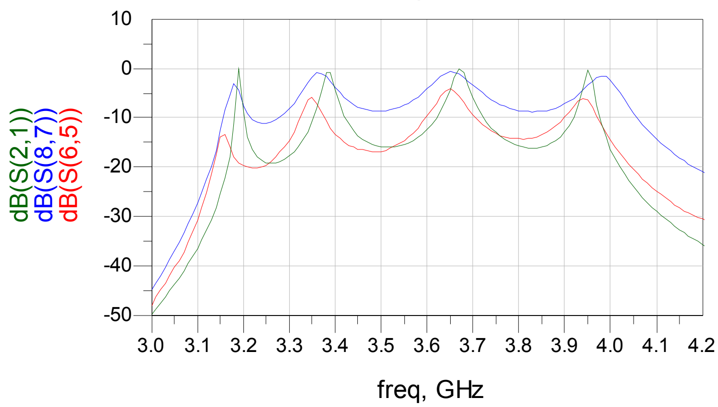


ADS ---

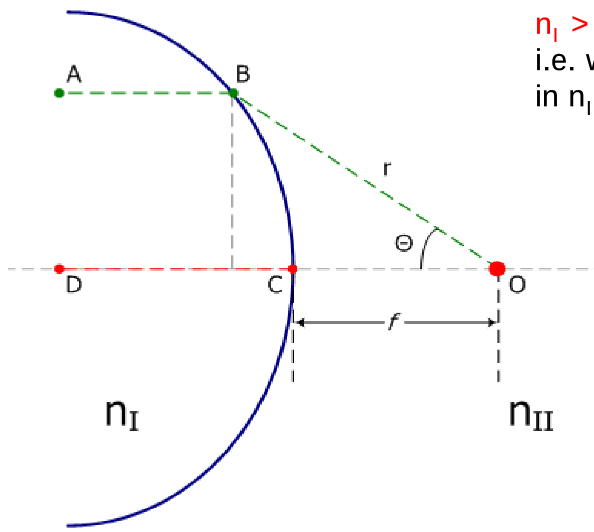
HFSS ---

Measured ---

$|S_{21}|$



Generalized Conical Meta-Interfaces



For this conic section to focus,
$n_I > n_{II}$
i.e. wave needs to be slower in n_I than in n_{II}

$$n_I \overline{AB} + n_{II} r = n_I \overline{DC} + n_{II} f$$

$$\Rightarrow \overline{DC} = \overline{AB} + (r\cos\theta - f)$$

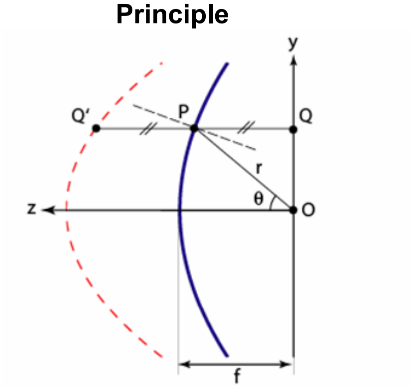
$$\ldots$$

$$r = \frac{f(n_I - n_{II})}{n_I \cos\theta - n_{II}}$$

$n_I > n_{II}$: Hyperbola
$n_I < n_{II}$: Ellipse
$n_I = -n_{II}$: Parabola

Paraboloidal "Refractor"

Principle



Real Parabola

$$z = f - \frac{y^2}{4f}$$

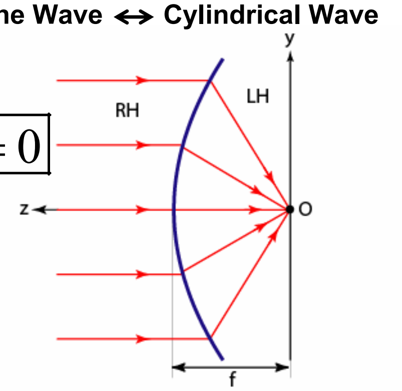
$$r = \frac{2f}{n_1 + n_2 \cos\theta}$$

Image Parabola

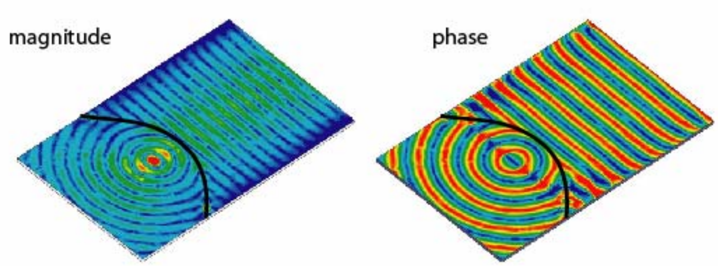
$$z = 2\left(f - \frac{y^2}{4f}\right)$$

Plane Wave ↔ Cylindrical Wave

$$\Gamma = 0$$



Effective Medium Full-Wave Demonstration



Mushroom Implementation